In Chapter 2, Section 5, we explored **direct variation**. Direct variation is modeled by the equation \( y = ax \) and we say that \( y \) varies directly with \( x \). The nonzero constant \( a \) is called the **constant of variation**. Notice, direct variation is a linear equation such that the \( y \)-intercept is \((0, a)\).

Graph the examples of direct variation and determine the constant of variation.

**a.** \( y = - \frac{1}{2} x \)

\[ a: \underline{-\frac{1}{2}} \]

**b.** \( \frac{y}{x} = 5 \) \( y = 5x \)

\[ a: \underline{5} \]

There are many **real-world applications** that model direct variation. For instance, we can say that distance \( d \) varies directly with time \( t \) at a constant rate \( r \), giving us the formula \( d = rt \) where \( d \) varies directly with \( t \) and the constant of variation is \( r \). We could list a simpler example, such as the more you study, the better grades you will get. Since both studying and grades are increasing (getting better), this models a direct variation.

Now, in Chapter 8, Section 1, we are looking at **inverse variation**. We say that two variables \( x \) and \( y \) show inverse variation if they can be modeled by the equation \( \frac{y}{x} = \frac{a}{x} \) when \( a \neq 0 \). The nonzero constant \( a \) is still called the **constant of variation**. We say that \( y \) varies inversely with \( x \) because as one variable increases, the other decreases. The equation \( y = \frac{a}{x} \) can be solved for \( a \) for an equivalent statement. Solve for \( a \) now: \( \underline{x} \frac{y}{x} = a \).

The graph of an inverse variation equation is the shape of a **rational equation** (where the variable is in the denominator).

Now you try to graph one: \( y = \frac{2}{x} \)

Make a table of positive and negative \( x \) values (but no zero) and plot the points.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>2</th>
<th>1</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-2</td>
<td>1</td>
<td>2</td>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>

An example of inverse variation is that the length \( l \) of a rectangle varies inversely with its width \( w \) given a constant area \( A \). This gives us the equation: \( l = \frac{A}{w} \). A simpler example is the more you slack off (increasing), the worse grade you will get (decreasing).

We are also looking at **joint variation**, which extends direct variation to get the following: \( z \) **varies jointly with \( x \) and \( y \) given that** \( z = axy \). This could be extended to even more quantities, such as \( p = aqrs \). Don’t ever forget your constant of variation in direct, inverse and joint variation equations.
A. Classify each equation as direct variation, inverse variation, joint variation, or none.

1. \( xy = 7 \) \( \frac{y}{x} = 7/x \) Inverse
2. \( y = x + 3 \) None
3. \( \frac{x}{y} = \frac{1}{4x} \) Direct
4. \( y = awx \) Joint

B. Write an inverse variation equation or joint variation equation.

1. The variables \( x \) and \( y \) vary inversely and \( y = 7 \) when \( x = 4 \). Write an equation that relates \( x \) and \( y \). Then find \( y \) when \( x = -2 \).

\[
\begin{align*}
y &= \frac{a}{x} \\
\Rightarrow \quad 7 &= \frac{a}{4} \therefore a &= 28 \\
y &= \frac{28}{x}
\end{align*}
\]

\[
\begin{align*}
y &= \frac{28}{x} \\
\Rightarrow \quad -2 &= \frac{a}{-2} \therefore a &= -14
\end{align*}
\]

2. The variables \( x \) and \( y \) vary inversely. The point \( \left( \frac{1}{2}, 12 \right) \) lies on the curve. Use the given values to write an equation relating \( x \) and \( y \). Then find \( y \) when \( x = 2 \).

\[
\begin{align*}
y &= \frac{a}{x} \\
\Rightarrow \quad 12 &= \frac{a}{\frac{1}{2}} \therefore a &= 6
\end{align*}
\]

\[
\begin{align*}
y &= \frac{6}{x} \\
\Rightarrow \quad \frac{1}{2} &= \frac{a}{\frac{1}{2}} \therefore a &= 3
\end{align*}
\]

3. The variable \( z \) varies jointly with \( x \) and \( y \). Also, \( z = -75 \) when \( x = 3 \) and \( y = -5 \). Write an equation that relates \( x \), \( y \), and \( z \). Then find \( z \) when \( x = 2 \) and \( y = 6 \).

\[
\begin{align*}
z &= axy \\
\Rightarrow \quad -75 &= a(3)(-5) \\
\therefore \quad a &= 5
\end{align*}
\]

\[
\begin{align*}
z &= axy \\
\Rightarrow \quad 60 &= a(2)(6) \\
\therefore \quad z &= 5(x)(y)
\end{align*}
\]

C. Example 3 in Book: Write an inverse variation Model. We use models to write equations for real-world applications. Read the information on p. 552 for MP3 songs. Show the work to answer the following questions:

- Write a model that gives the number \( n \) of songs that will fit on the MP3 player as a function of the average song size \( s \) (in megabytes).

\[
\begin{align*}
n &= \frac{a}{s} \\
2.5 &= \frac{a}{\frac{1}{2}} \therefore a &= 10,000
\end{align*}
\]

- Make a table showing the number of songs that will fit on the MP3 player if the average size of a song is 2 MB, 2.5 MB, 3 MB and 5 MB. What happens to the number of songs as the average song size increases?

<table>
<thead>
<tr>
<th>( n )</th>
<th>5,000</th>
<th>4,000</th>
<th>3,333</th>
<th>2,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>2</td>
<td>2.5</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

\( \frac{\text{As } s \text{ increases, number of songs decreases.}}{\text{}} \)

D. Example 4 in Book: Following example 4 from the book, try this similar example.

The table below compares the current \( I \) (in milliamps) with the resistance \( R \) (in ohms) for several electrical circuits. Determine if the table shows inverse variation. If so, write a model that gives \( R \) as a function of \( f \).

<table>
<thead>
<tr>
<th>Current (milliamps) ( I )</th>
<th>7.4</th>
<th>8.9</th>
<th>12.1</th>
<th>17.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance (ohms) ( R )</td>
<td>1200</td>
<td>1000</td>
<td>750</td>
<td>500</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
1R &= 8900 \\
7.4(1200) &= 8880 \\
8.9(1000) &= 8900 \\
12.1(750) &= 9075 \\
17.9(500) &= 8950
\end{align*}
\]

\( 1R \approx 8900 \)

E. Write an equation for each relationship.

1. \( y \) varies inversely with the square of \( x \).
   \( y = \frac{a}{x^2} \)

2. \( x \) varies jointly with \( t \) and \( r \) and inversely with \( s \).
   \( x = \frac{atr}{s} \)

3. \( p \) varies jointly with \( q, r \) and \( s \), and inversely with the cube of \( t \).
   \( p = \frac{aqrst}{t^3} \)

4. \( x \) varies directly with \( y \) and inversely with \( z \).
   \( x = \frac{ay}{z} \)